B.Sc. Part-I

Paper-I

Theory of Relativity

Dr. Shiva Kant Mishra
Dept of Physics HDJC

Relativistic Doppler Effect :-

25.6

The relativistic Doppler effect is the change in frequency of light, caused by the relative motion of the source and the observer (as in the classical Doppler effect), when taking into account effects described by the special theory of relativity.

The relativistic Doppler effect is different from the non-relativistic Doppler effect as the equations include the time dilation effect of special relativity and do not involve the medium of propagation as a reference point. They describe the total difference in observed frequencies and possess the required Lorentz symmetry.

Astronomers know of three sources of redshift/blueshift: Doppler shift, Gravitational shifts and cosmological expansion

Relativistic longitudinal Doppler effect :-

Relativistic Doppler shift for the longitudinal case, with source and receiver moving directly towards or away from each other, is often derived as if it were the classical phenomenon, but modified by the addition of a time dilation term. ~~This is the approach employed in first year~~

Expt. No. ______ Page No. ______

Following this approach towards deriving the relativistic longitudinal Doppler effect, assume the receiver and the source are moving away from each other with a relative speed v as measured by an observer on the receiver or the source ($\in$ the sign convention adopted here is that v is -ve if the receiver and the source are moving towards each other).

Consider the problem in the reference frame of the source.

Suppose one wavefront arrives at the receiver. The next wavefront is then at a distance $\lambda_s = c/f_s$ away from the receiver (where λ_s is the wavelength, f_s is the frequency of the waves that the source emits, and c is the speed of light).

The wavefront moves with speed c, but at the same time the receiver moves away with speed v during a time $t_{r,s}$ which is the period of light waves impinging on the receiver, as observed in the frame of the source.

So,

$$\lambda_s + v t_{r,s} = c t_{r,s} \Leftrightarrow \lambda_s = c t_{r,s}(1 - v/c) \Leftrightarrow$$

$$t_{r,s} = \frac{1}{f_s(1-\beta)},$$

Teacher's Signature ______

where $\beta = v/c$ is the speed of the receiver in terms of the speed of light. The corresponding $f_{r,s}$, the frequency of at which wave fronts impinge on the receiver in the source's frame is,

$$f_{r,s} = 1/t_{r,s} = f_s(1-\beta).$$

Thus far the eqn's have been identical to those of the classical Doppler effect with a stationary source and a moving receiver.

However, due to relativistic effects, clocks on the receiver are ~~the~~ time dilated relative to clocks at the source: $t_r = t_{r,s}/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor. In order to know which time is dilated, we recall that $t_{r,s}$ is the time in the frame in which the source is at rest. The receiver will measure the received frequency to be

$$f_r = f_{r,s}\,\gamma = \frac{1-\beta}{\sqrt{1-\beta^2}} f_s = \sqrt{\frac{1-\beta}{1+\beta}}\, f_s \quad \text{--- (1)}$$

The ratio $\dfrac{f_s}{f_r} = \sqrt{\dfrac{1+\beta}{1-\beta}}$ is called the Doppler factor ~~of~~ the source relative to the receiver.

The corresponding wavelengths are related by

$$\frac{\lambda_r}{\lambda_s} = \frac{f_s}{f_r} = \sqrt{\frac{1+\beta}{1-\beta}} \quad \text{--- (2)}$$

Expt. No. ______ Page No. ______

Identical expressions for relativistic Doppler shift are obtained when performing the analysis in the reference frame of the receiver with a moving source. This matches up with the expectations of the principle of relativity, which dictates that the result can not depend on which object is considered to be the one at rest. In contrast, the classic non-relativistic Doppler effect is dependent on whether it is the source or the receiver that is stationary with respect to the medium.